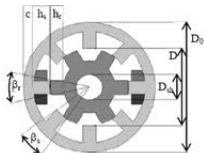


Model elicitation for synchronous machine

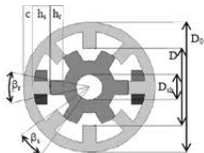
Václav Šmíd

October 29, 2012

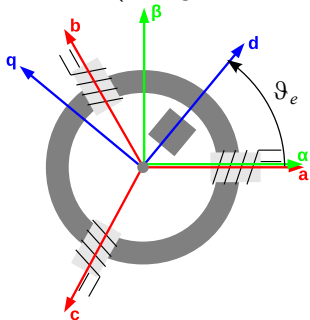
Permanent Magnet Synchronous Machine



Permanent Magnet Synchronous Machine



Variables (voltages, currents) treated as vectors in 2D space.



Motivation - sensorless control

Model of the drive in $\alpha - \beta$

$$\omega_{t+1} = \omega_t + \frac{k_p p_p^2 \Psi_{pm}}{J} \Delta t (i_{\beta,t} \cos(\vartheta) - i_{\alpha,t} \sin(\vartheta)) - \frac{p_p \Delta t}{J} T_L,$$
$$\vartheta_{t+1} = \vartheta_t + \Delta t \omega_{me}.$$

Observations

$$i_{\alpha,t+1} = \left(1 - \frac{R_s}{L_s} \Delta t\right) i_{\alpha,t} + \frac{\Psi_{PM} \Delta t}{L_s} \omega_{me} \sin \vartheta + \Delta t \frac{u_{\alpha,t}}{L_s},$$
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At $\omega = 0$, ϑ_e is unobservable.

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At $\omega = 0$, ϑ_e is unobservable.

Working estimation technique: High-frequency injections with phase-lock-loop.

Physical model of the PMSM drive

Observation equation for currents in dq coordinate system:

$$\begin{aligned}i_{d,t+1} &= \left(1 - \frac{R_s}{L_{s,d}} \Delta t\right) i_{d,t} + i_{q,t} \omega_t \Delta t + u_{d,t} \frac{\Delta t}{L_{s,d}}, \\i_{q,t+1} &= \left(1 - \frac{R_s}{L_{s,q}} \Delta t\right) i_{q,t} - \left(\frac{\psi_{pm}}{L_{s,q}} + i_{d,t}\right) \Delta t \omega_t + u_{q,t} \frac{\Delta t}{L_{s,q}},\end{aligned}\quad (1)$$

Inductances in d and q axis slightly differ – cca 2-5%.

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Inductances in d and q axis slightly differ – cca 2-5%.

- ▶ Detailed measurement of inductances is difficult
- ▶ Restructuring the model:

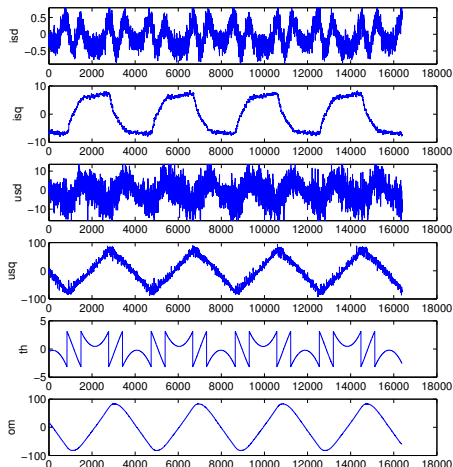
$$\begin{aligned}u_d &= [L_d, R_s, L_q] \left[\frac{i_{d,t} - i_{d,t-1}}{\Delta t}, i_d, -i_q \omega_{me} \right] + \xi_{ud}, \\u_q &= \underbrace{[L_d, R_s, L_q, \Psi_{pm}]}_{\theta} \left[\underbrace{i_d \omega_{me}, i_q, \frac{i_{q,t} - i_{q,t-1}}{\Delta t}, \omega_{me}}_{\psi} \right] + \xi_{uq},\end{aligned}$$

Recorded data

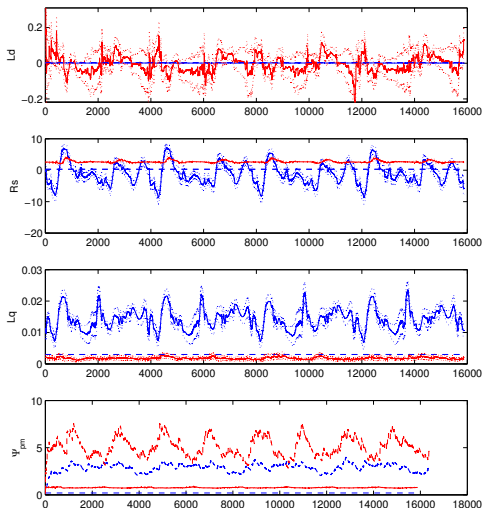
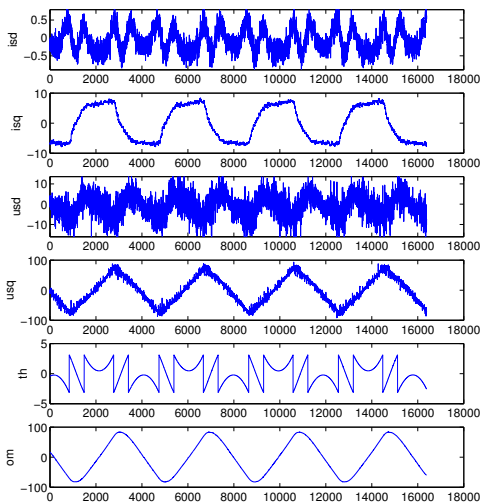
- ▶ triangular profile of ω to have maximum range,
- ▶ excitation of the inputs by square HF injections

$$u_t = u_{t,opt} + 7V \cdot 1kHz \text{ squares}$$

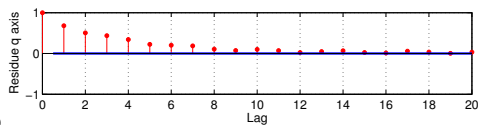
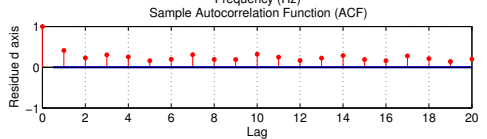
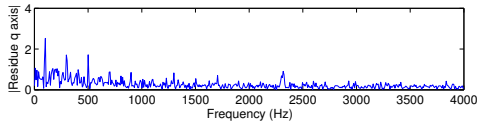
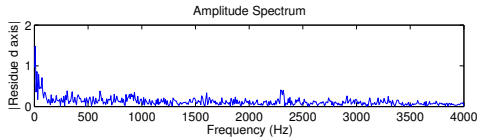
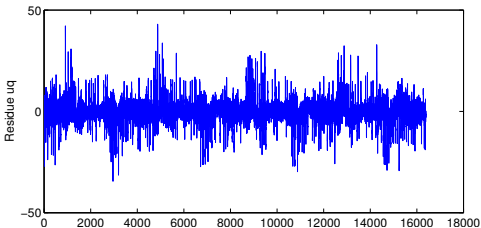
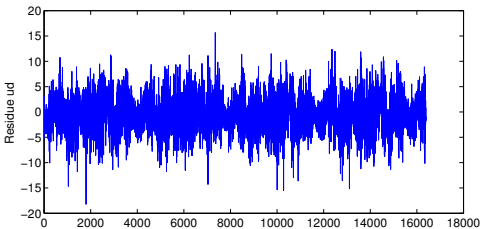
- ▶ low currents in i_d axis



Estimation with recorded data



Residues

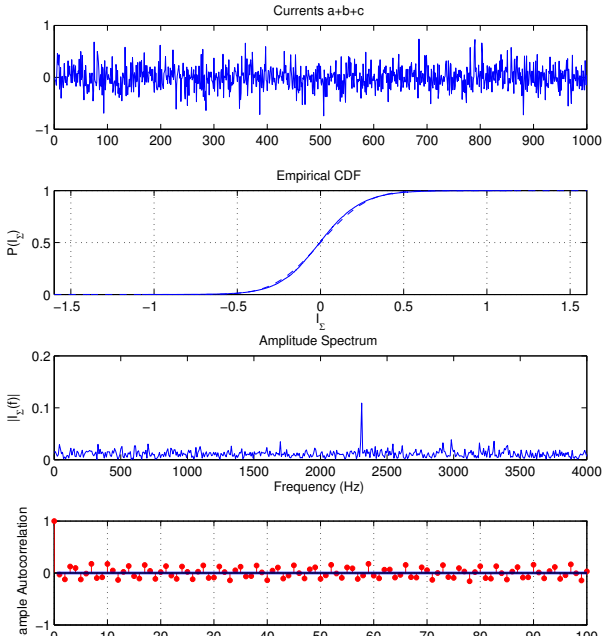


Is the measurement noise Gaussian?

Unique property.
Law of physics:

$$i_a + i_b + i_c = 0.$$

Gaussian cdf.



Different parametrization

Model

$$i_{d,t+1} = \left(1 - \frac{R_s}{L_{s,d}} \Delta t\right) i_{d,t} + i_{q,t} \omega_t \Delta t + u_{d,t} \frac{\Delta t}{L_{s,d}},$$

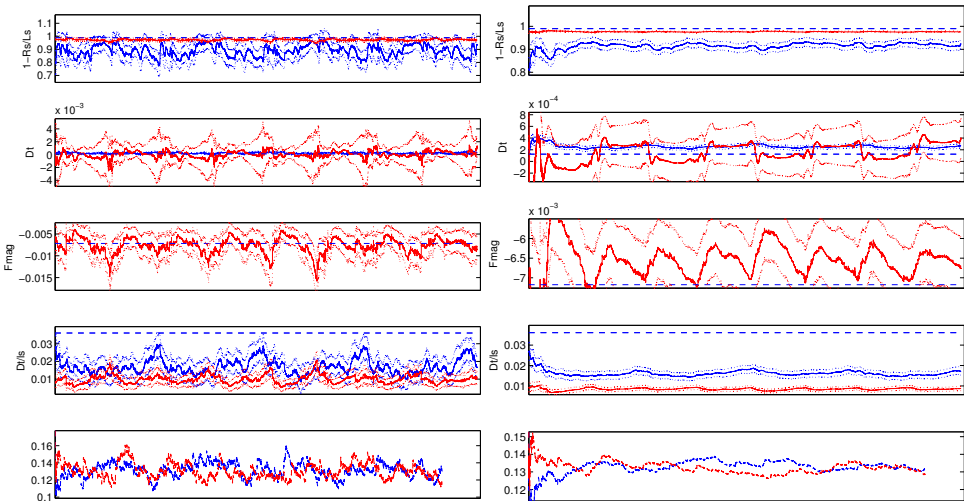
$$i_{q,t+1} = \left(1 - \frac{R_s}{L_{s,q}} \Delta t\right) i_{q,t} - \left(\frac{\psi_{pm}}{L_{s,q}} + i_{d,t}\right) \Delta t \omega_t + u_{q,t} \frac{\Delta t}{L_{s,q}},$$

is

$$\begin{aligned} i_{d,t+1} &= [a_d, b_d, c_d][i_{d,t}, i_{q,t} \omega_t, u_{d,t}], \\ i_{q,t+1} &= \underbrace{[a_q, b_q, f_q, c_q]}_{\theta} \underbrace{[i_{q,t}, \omega_t, i_{d,t} \omega_t, u_{q,t}]}_{\psi}, \end{aligned}$$

- Parameters have no physical meaning,
- + Directly applicable in a state space estimator.

Results of estimation, $\lambda = 0.995$, $\lambda = 0.9995$



Dead times and voltage drops

- ▶ The input voltage is not exact!
- ▶ It is transformed by PWM unit.
 - ▶ voltage drops,
 - ▶ dead times
- ▶ Model

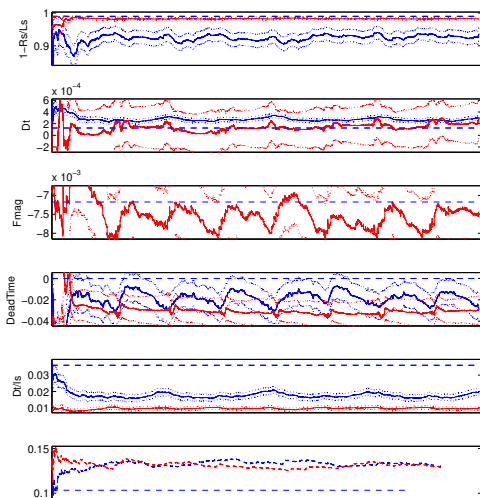
$$u_{x,true} = u_{x,t} - u_{dead_time} \operatorname{sgn}(i_x).$$

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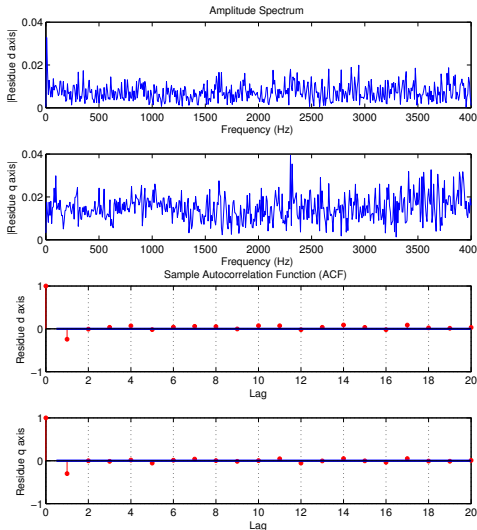
$$u_{x,true} = u_{x,t} - u_{dead_time} \text{sgn}(i_x).$$

- ▶ u_{dead_time} estimated stationary,
- ▶ Estimated sampling time Δt matches its true value in the q axis!



Residues

- ▶ Dominant frequency on the residues on the q is the measurement peak!
- ▶ Dominant frequency on the d axis is 10Hz.
- ▶ On both axis, there is still negative correlation with 1-step delayed value.
 - ▶ suggests missing element in the equations



Automatic search - hypothesis testing

Correlation of the residue with its previous value suggests a missing element.

Existing regressands in q axis:

$$\psi = [i_{q,t}, \omega_t, i_{d,t}\omega_t, u_{q,t}, \text{sgn}(i_q)],$$

Candidates for extension:

additional element	none	1	i_d	u_d	$i_d u_d$	$\text{sgn}(i_d)$	$i_{q,t-2}$
marginal likelihood	46%	1%	11%	10%	17%	13%	1%
autocorrelation -1	-0.302	-0.301	-0.300	-0.301	-0.301	-0.301	-0.08
autocorrelation -2	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.16

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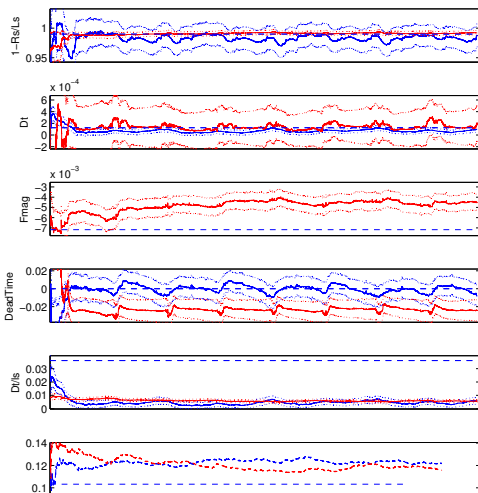
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ARMA?

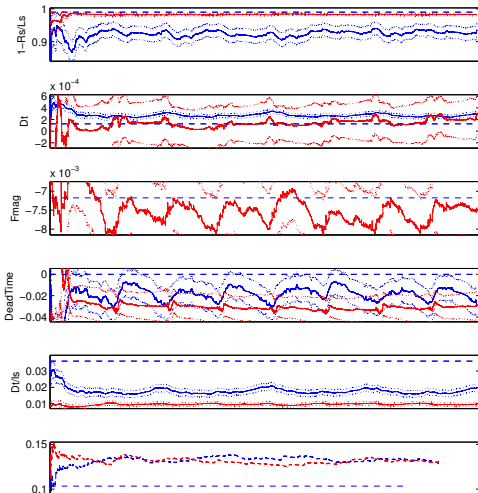
additional element	none	ϵ_{t-1}	ϵ_{t-1} , no $\text{sgn}(i_d)$
marginal likelihood	0%	100.00%	0%
autocorrelation -1	-0.302	-0.124	-0.104
autocorrelation -2	-0.01	-0.08	-0.04

Parameters of the ARMA model, $\lambda = 0.9995$

ARMA

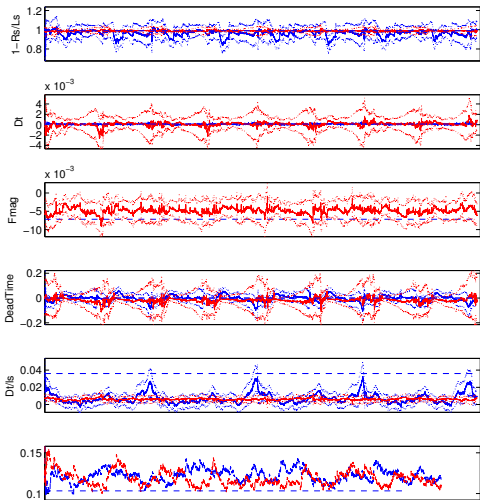


AR

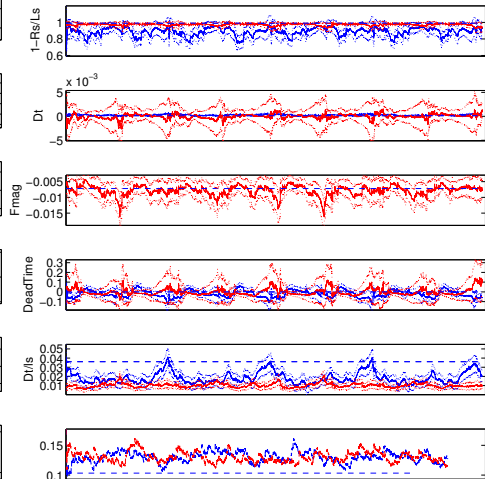


Parameters of the ARMA model, $\lambda = 0.995$

ARMA



AR



Conclusion

- ▶ Model elicitation from real data is a demanding task
 - ▶ experiment design,
 - ▶ sensor calibration!
- ▶ Poor excitation in the d axis can be improved by inefficient control (exploration),
- ▶ PMSM drive model:
 - ▶ is it an ARMA process?
$$\epsilon_t - 0.5\epsilon_{t-1}$$
 - ▶ can ARMA model replace dead-times and voltage drops?